Videoaufzeichnung 1

→ Homepage Feldmeier

Verzeichnis theomech2021

dort: vorl01.mp4 ... vorl99.mp4

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Übungen sind Prüfungsvorleistung

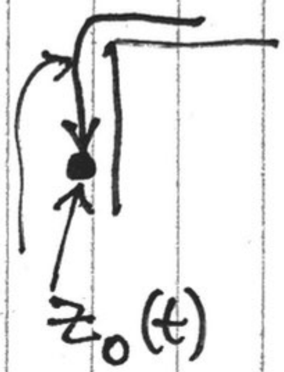
Klausur I Mo 26.7. $15^{\underline{00}}$ – $17^{\underline{00}}$ ... $18^{\underline{00}}$ ← gr Ph. gr ch. (online)

II Mi 20.10.

Bücher: F. / Nolting / Greiner / Fließbach / Kuypers / Landau-Lifschitz

I Kinematik $\vec{F} = 0$

II Newton $<$ Kepler / Feder $\vec{F} = m\vec{a}$



III Lagrange $\frac{\partial L}{\partial q_i} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} = 0$

IV Hamilton $\dot{q}_i = \frac{\partial H}{\partial p_i}$

$\dot{p}_i = -\frac{\partial H}{\partial q_i}$

Bild

KAM Theorem

Themen

V Drehgruppe f. starren Körper

Erinnerung Vorkurs

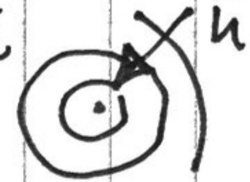
1) Gradient

$$df := \nabla f \cdot \vec{dr}$$

$f = f(x, y, z)$

$f(r, \vartheta, \varphi)$

$$\nabla f = \frac{\partial f}{\partial n}\,\hat{n}$$



$$\nabla f = \begin{pmatrix} f_x \\ f_y \\ f_z \end{pmatrix}$$

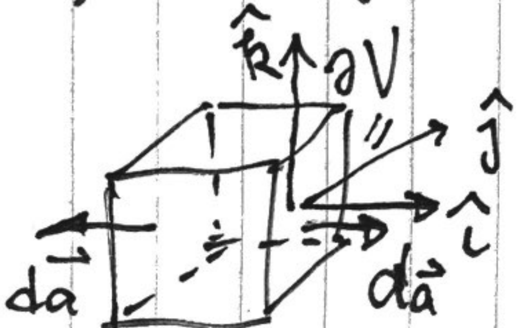
$u \sim \frac{1}{r}$

$f_x : \frac{\partial f}{\partial x}$ aber auch $\vec{v} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}$

2) Divergenz

$$dV\, \nabla \cdot \vec{v} := \oint_{\partial dV} \vec{da} \cdot \vec{v}$$

$$\int_V dV\, \nabla \cdot \vec{v} = \oint_{\partial V} \vec{da} \cdot \vec{v}$$ Satz von Gauß



$\hat{i}$

$$\rightsquigarrow \nabla \cdot \vec{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

sei $\nabla = \begin{pmatrix} \partial/\partial x \\ \partial/\partial y \\ \partial/\partial z \end{pmatrix}$ nabla

Jedes Vektorfeld ist durch seinen rotor & divergenz „eindeutig" bestimmt (bis auf Lsg der Laplacegl.)

$$\nabla f = \begin{pmatrix} \partial f/\partial x \\ \partial f/\partial y \\ \partial f/\partial z \end{pmatrix} = \text{grad } f$$

$$\nabla \cdot \vec{v} = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \cdot \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

d v x nach dx

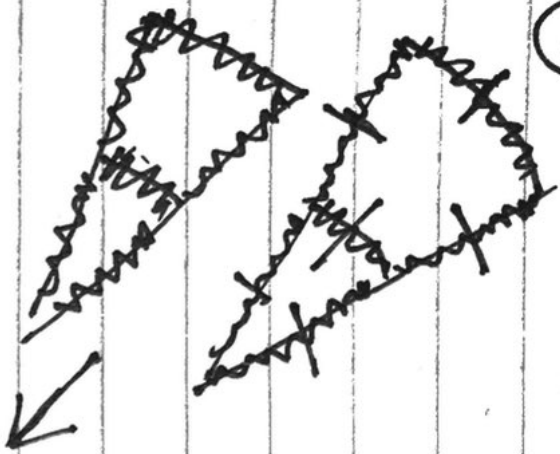
Helmholtz

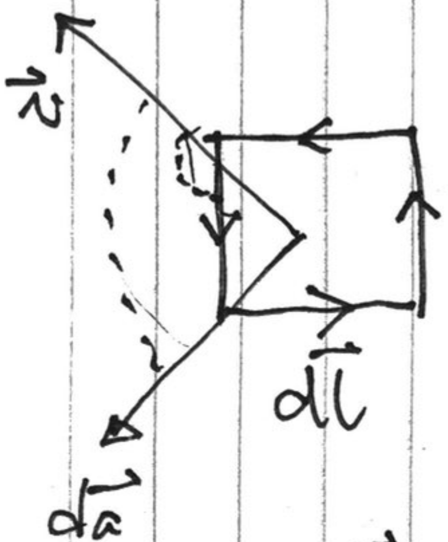
rotor / curl

$\nabla \times \vec{v} =$ ⇝ gibt Vektor

3) Rotor

$$d\vec{a}\cdot\nabla\times\vec{v} := \oint_{\partial d\vec{a}} d\vec{l}\cdot\vec{v} \qquad \text{Def}$$

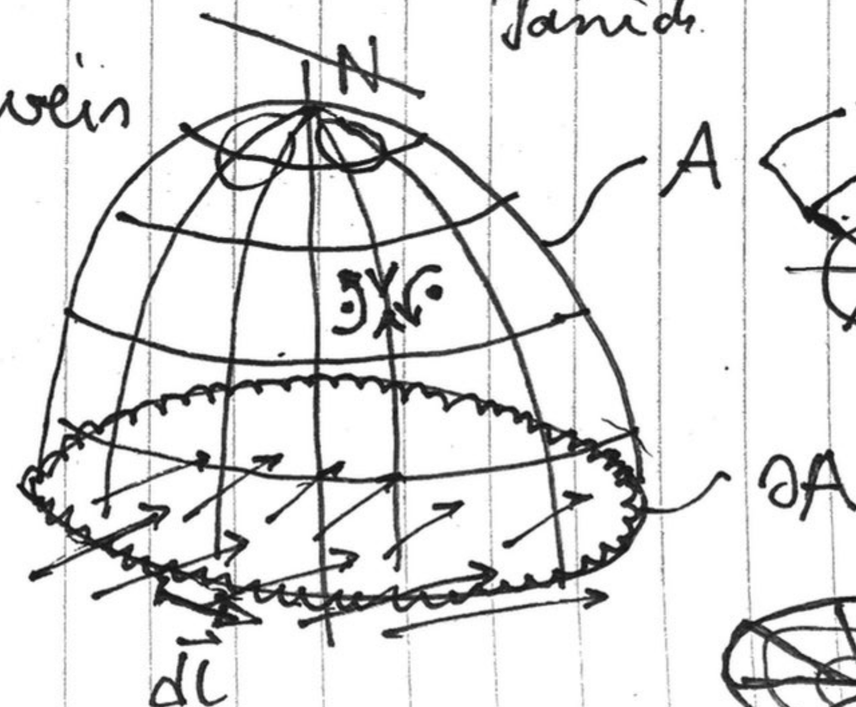


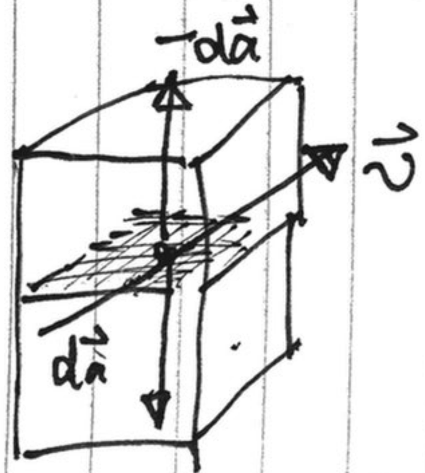


$$\int_A d\vec{a}\cdot\nabla\times\vec{v} = \oint_{\partial A} d\vec{l}\cdot\vec{v}$$

Jänich

Beweis



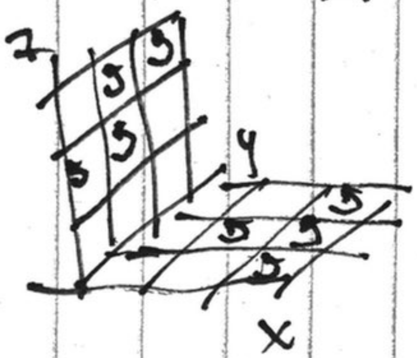


$$\vec{v}\cdot d\vec{a} = -\vec{v}\cdot(-d\vec{a})$$

8

1) Def rot $$d\vec{a}\cdot\nabla\times\vec{v} = \oint_{\partial d\vec{a}} d\vec{l}\cdot\vec{v}$$

2) Satz von Stokes $$\int_A d\vec{a}\cdot\nabla\times\vec{v} = \int_{\partial A} d\vec{l}\cdot\vec{v}$$ mit Maschen integral

3) 

$$\underbrace{\nabla\times\vec{v}}_{\text{rot }\vec{v}} = \nabla\times\vec{v} = \begin{pmatrix}\frac{\partial}{\partial x}\\ \frac{\partial}{\partial y}\\ \frac{\partial}{\partial z}\end{pmatrix}\times\begin{pmatrix}v_x\\ v_y\\ v_z\end{pmatrix}$$

$\nabla\cdot\nabla$

$\nabla\times\nabla\times$

$\nabla\nabla\cdot$

NEU

rot grad f

geht auch mit Integralsätzen

$$\begin{pmatrix}\partial/\partial x\\ \partial/\partial y\\ \partial/\partial z\end{pmatrix}\times\begin{pmatrix}\partial f/\partial x\\ \partial f/\partial y\\ \partial f/\partial z\end{pmatrix} = \begin{pmatrix}\frac{\partial^2 f}{\partial y\,\partial z} - \frac{\partial^2 f}{\partial z\,\partial y}\\ \dots - \dots\\ \dots - \dots\end{pmatrix} \equiv 0$$