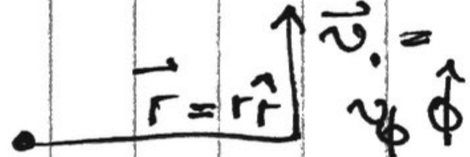
Effektives Potential

Pot. Grav: $\Phi(\vec r) = -\frac{\gamma}{r}$ $\quad(\gamma = GM)$

denn $\nabla \frac{1}{r} = \frac{d(1/r)}{dr}\,\hat r = -\frac{\hat r}{r^2}$

$$\vec F = -m\nabla\Phi = - \; - \; - \; \underbrace{\gamma m}_{\Gamma}\,\frac{1}{r^2}\,\hat r$$



Betrachte $\frac{E}{m} = e$, $\frac{L}{m} = l$, usw

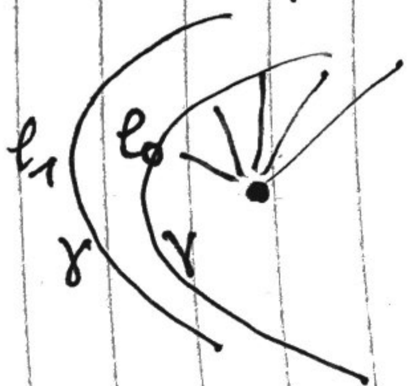
$l = r v_\phi = \text{const}$

Energiesatz in Polkoor.

$v_\phi^2 = \frac{l^2}{r^2}$

$$\frac{1}{2} v_r^2 + \frac{1}{2} v_\phi^2 - \frac{\gamma}{r} = e$$

$$\frac{1}{2} v_r^2 + \underbrace{\frac{l^2}{2r^2} - \frac{\gamma}{r}}_{V_{eff}} = e$$



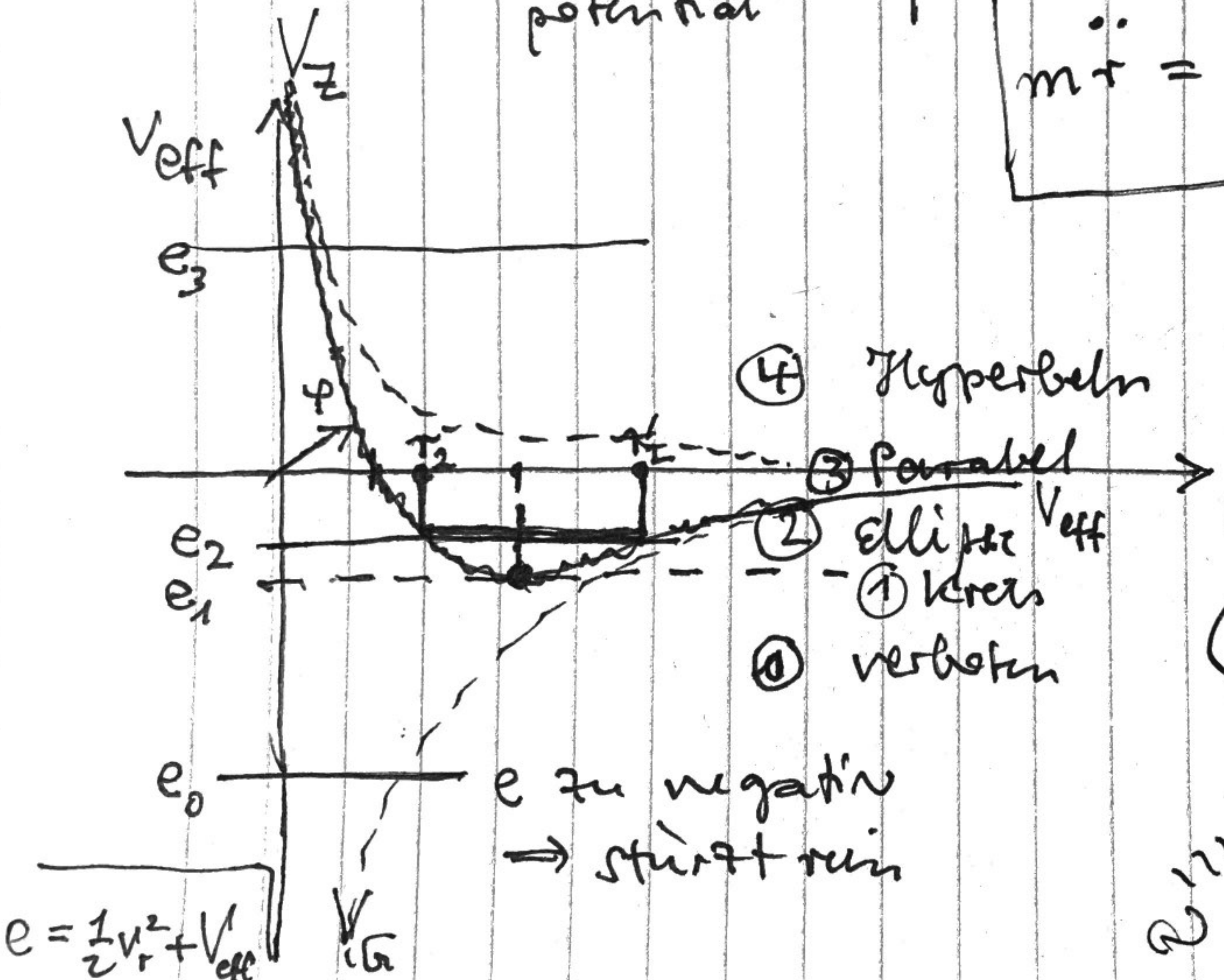
$$V_{eff} = \underbrace{\frac{\ell^2}{2r^2}}_{\text{zentrifugal potential}} - \frac{\gamma}{r}$$

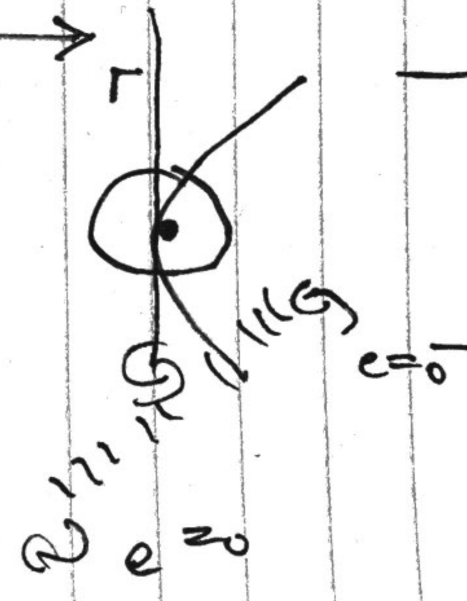
Nachweis

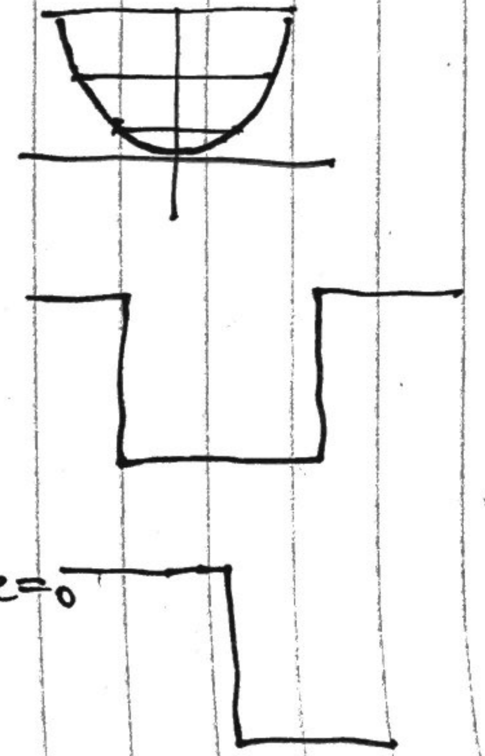
$$a_Z = \frac{v_\phi^2}{r} = \frac{r^2 v_\phi^2}{r^3} = \frac{\ell^2}{r^3} = -\frac{d}{dr}\frac{\ell^2}{2r^2}$$

$$= -\frac{d}{dr} V_Z$$

$$\boxed{m\ddot{r} = -\frac{dV_{eff}}{dr}}$$

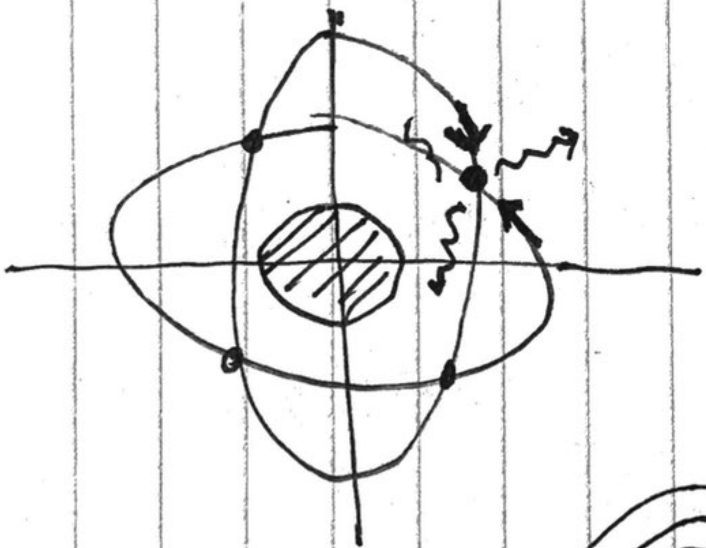






negative Gesamt (potential) energie

= Bindungszustand



$\Delta e = 0$

$\cdot \Delta t = \infty \geq \hbar$

Balbus - Hawley - Inst

→ MRI Magnetorotational Instability

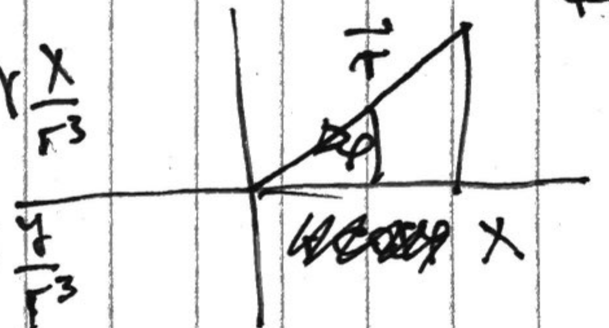
Keplerproblem

mit Schwerpkt. fest. $\ddot{\vec{r}} = -\frac{\gamma}{r^2}\hat{r} = -\gamma\frac{\vec{r}}{r^3}$

wir wissen ! Ebene $\to$ xy

$$\ddot{x} = \cancel{F_x} = -\gamma\frac{x}{\sqrt{x^2+y^2}^3} = -\gamma\frac{x}{r^3}$$

$$\ddot{y} = \cancel{F_y} = -\gamma\frac{y}{\sqrt{x^2+y^2}^3} = -\gamma\frac{y}{r^3}$$



$$\vec{a} = -\gamma\begin{pmatrix} x/r^3 \\ y/r^3 \end{pmatrix} = -\gamma\frac{\vec{r}}{r^3}$$

$$\boxed{\begin{aligned}\ddot{x} &= -\gamma\, x/r^3 \quad \cdot y \\ \ddot{y} &= -\gamma\, y/r^3 \quad \cdot x\end{aligned}}$$

(I), (II), (III), (IV)

$$\vec{l} = \begin{pmatrix} x \\ y \end{pmatrix} \times \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix}$$

$$\ddot{x}y - \ddot{y}x = 0 = \vec{r} \times \ddot{\vec{r}}$$

$$= \overset{\bullet}{\overbrace{\dot{x}y - \dot{y}x}}$$ d.h. $\dot{x}y - x\dot{y} = \text{const}$

(I): $l = \text{const}$