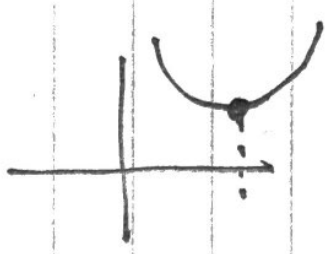
$f : \vec{x} \mapsto \vec{y}$



$J : f \mapsto \mathbb{R}/\mathbb{C}$

$g \circ f(x) : x \overset{f}{\mapsto} y \overset{g}{\mapsto} z$

$J[f] : \quad f \mapsto x$

$\uparrow$

J Riemann

$\uparrow$

x

$$M[f] = \max_{x \in ?} f(x)$$

Mechanik: finde $\vec{r}(t)$

$x_1(t), x_2(t), x_3(t), \dots, x_g(t)$

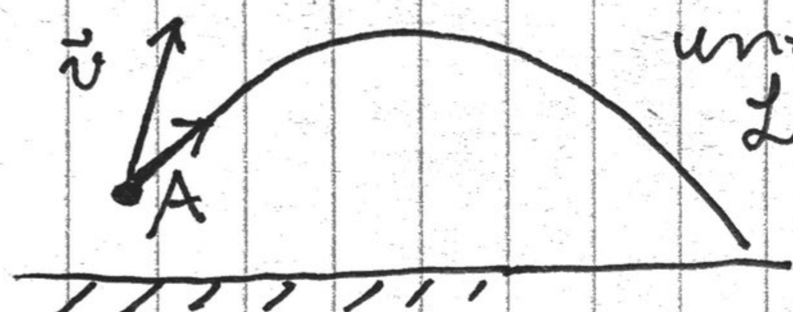
Suche diese Fkt $x_1, \dots, x_g$ mittels minimierter Funktionale

$$0 \overset{!}{=} \delta S = \underbrace{\delta \int dt}\, \mathcal{L} = \delta \int dt\, \mathcal{L}(q_1, \dots, q_n, \dot{q}_1, \dots, \dot{q}_n, t)$$

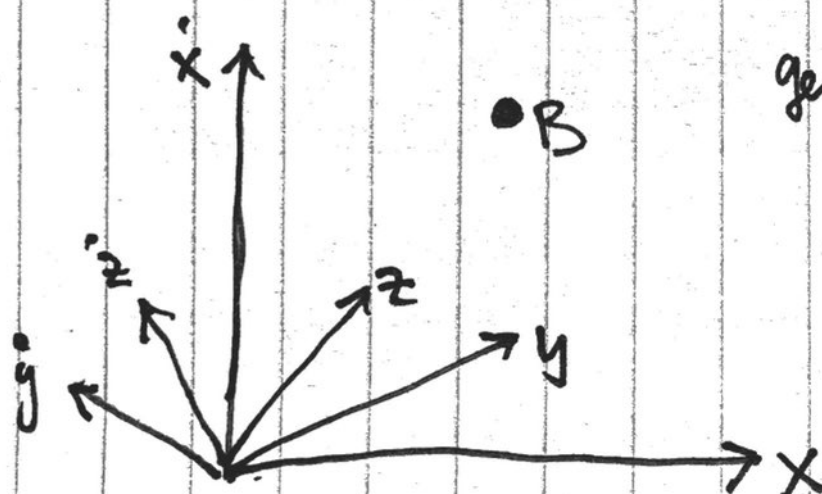
n = Zahl der Freiheitsgrade

Picard Lindelöf

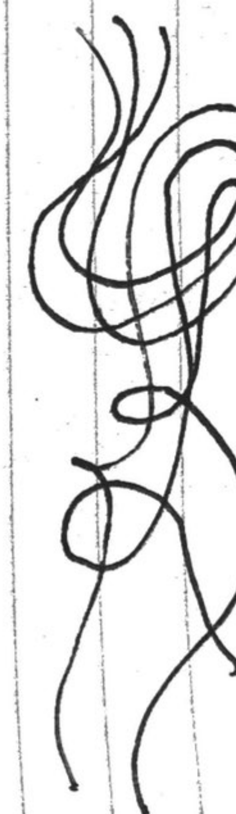
ab jetzt Phasenraum

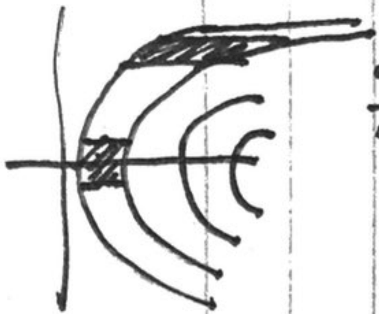


unendl. viele Lsg./Bahnen durch • A



genau eine Bahn/Lsg durch • B





$\ddot{\vec r} = \ddot{\vec r}(\vec r, \dot{\vec r})$ ist DGL 2. Ordnung, erfordert also im $\mathbb{R}^3$ 6 Anfangsbedg.

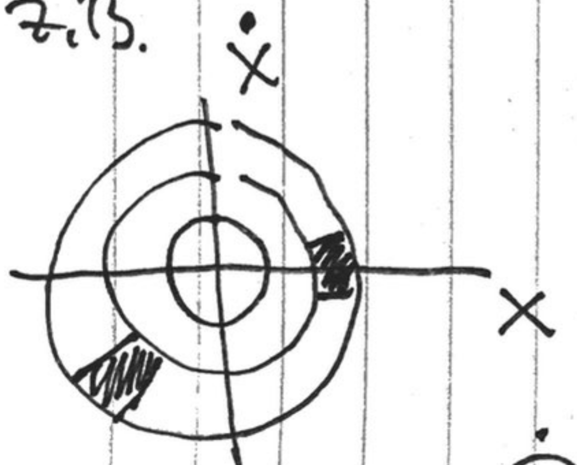
wichtigste Eigenschaft Phasenraum

1) Eindeutigkeit der Bahn durch Punkt

Blätterung/ Faserung

2) Liouville Satz: z.B. $n = 2$, $\dim(Ph) = 4$

z.B.



$\ddot x = -x \mid \dot x$

$\dot{(x^2)} + \dot{(\dot x^2)} = 0 \to x^2 + \dot x^2 = \text{const}$

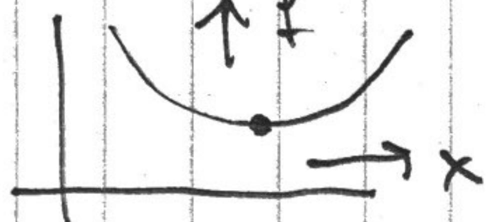
$V_0 = V_1$

extrem. Operator

Funktional

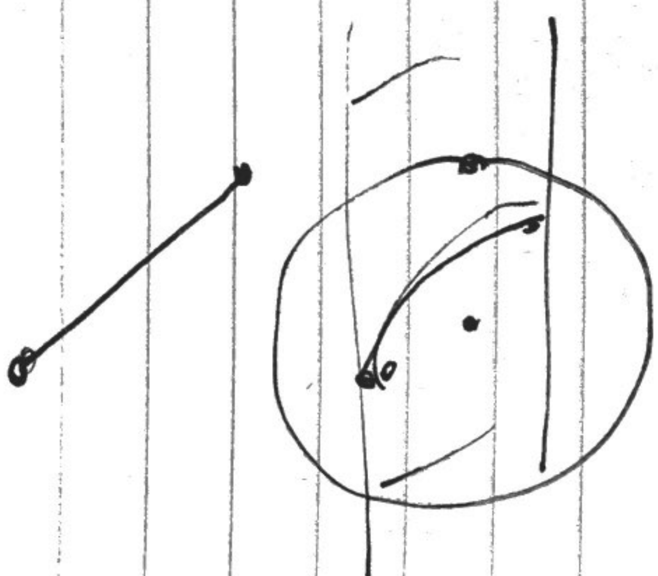
~~IM~~ Variationsrechnung $\delta\left[\overbrace{\int dt\, \mathcal{L}(q,\dot{q})}^{\text{Funktional}}\right] = 0$ hat Lsg. $q(t)$ für was die N II wenn $\mathcal{L} = \mathcal{L}_{mech}$

Analysis $d\, f(x) = 0$ hat Lsg. x_{min}



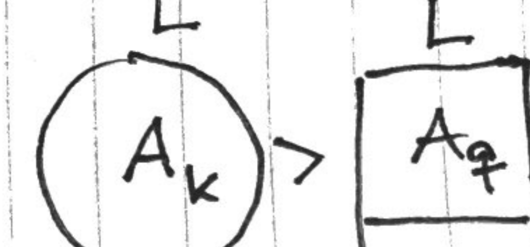
$$\boxed{\mathcal{L} = T - V}$$

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$



Brachistochrone

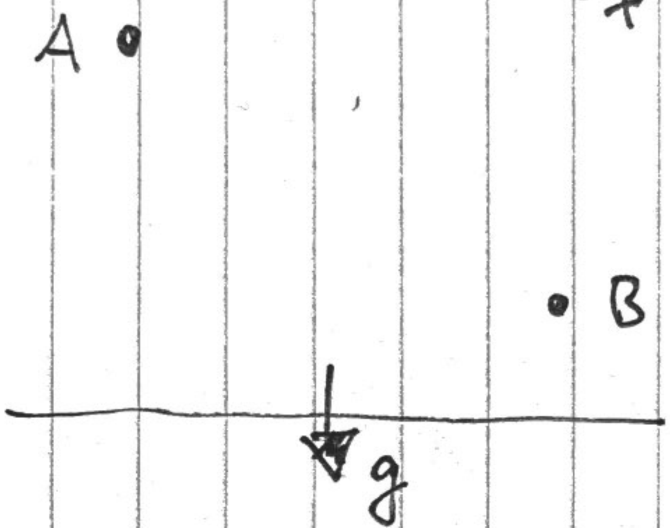
Bernoulli



$$\delta \int dt \; \mathcal{L}(q, \dot q, \ddot q)$$

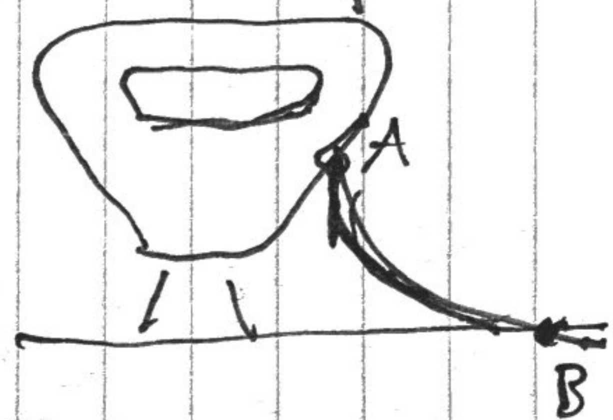
Bernoulli

$$\frac{\partial \mathcal{L}}{\partial q} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot q} + \frac{d^2}{dt^2}\frac{\partial \mathcal{L}}{\partial \ddot q} = 0$$

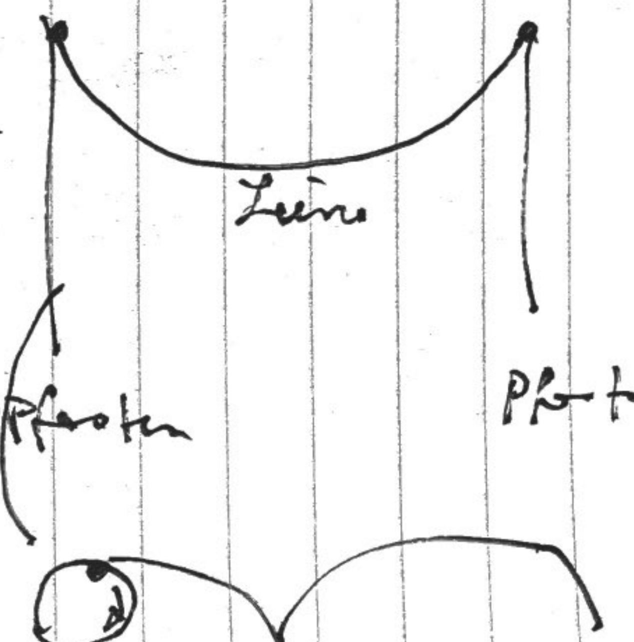


$q(x, t)$

$q', \dot q$



$$\delta \int dt \; \mathcal{L}(q, \dot q) = 0$$



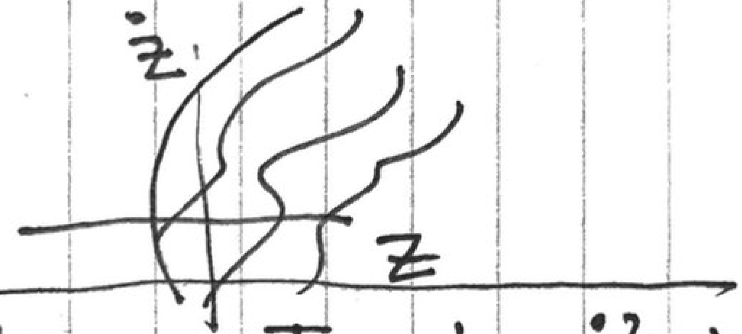
$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot q_i} - \frac{\partial \mathcal{L}}{\partial q_i} = 0 \quad \Bigg| \quad \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu A_\nu)} - \frac{\partial \mathcal{L}}{\partial A_\nu} = 0$$

Pfosten

Euler-Lagrange-Glgen

$$\frac{d}{dt}\frac{\partial L}{\partial \dot z} - \frac{\partial L}{\partial z} = 0 \quad \text{ist} \quad ma - mg = 0$$

für Grav. $L = \frac{m}{2}\dot z^2 - mgz$



$$\frac{\partial L}{\partial \dot z} = m\dot z$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot z}\right) = m\ddot z$$

$$\frac{\partial L}{\partial z} = -mg$$

für harm. Osz. $T = \frac{1}{2}m\dot x^2$, $V = \frac{1}{2}kx^2$

$$L = \frac{1}{2}m\dot x^2 - \frac{1}{2}mx^2$$

$$\left.\begin{aligned} \frac{d}{dt}\frac{\partial L}{\partial \dot x} &= m\ddot x \\ \frac{\partial L}{\partial x} &= kx \end{aligned}\right\} \quad m\ddot x - kx = 0$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot z} - \frac{\partial L}{\partial z} = m\ddot z + mg = 0$$