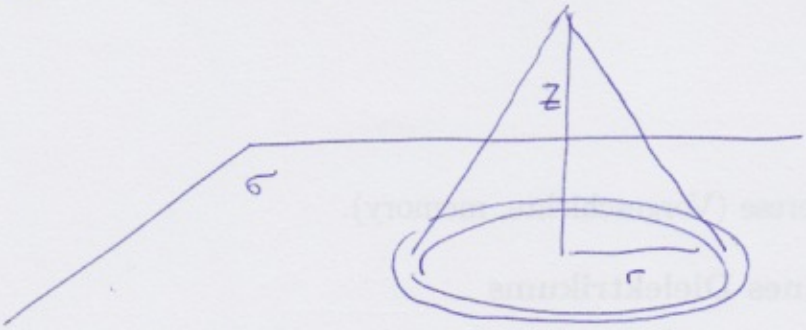
1 a



Jeder Punkt auf dem Kreisring gibt gleichen Beitrag zum Potential. Also

$$\Phi(z) = \frac{1}{4\pi\varepsilon_0} \int_0^\infty \frac{\overbrace{\sigma\, 2\pi r\, dr}^{= dq}}{\sqrt{r^2+z^2}}$$

$$= \frac{\sigma}{2\varepsilon_0} \int_0^\infty dr \, \frac{r}{R} \quad (\text{mit } R^2 = r^2 + z^2)$$

$$\stackrel{\text{Angabe}}{=} \frac{\sigma}{2\varepsilon_0} R \Big|_0^{r=\infty} = \frac{\sigma}{2\varepsilon_0} \sqrt{r^2+z^2}\Big|_{r=0}^{\infty}$$

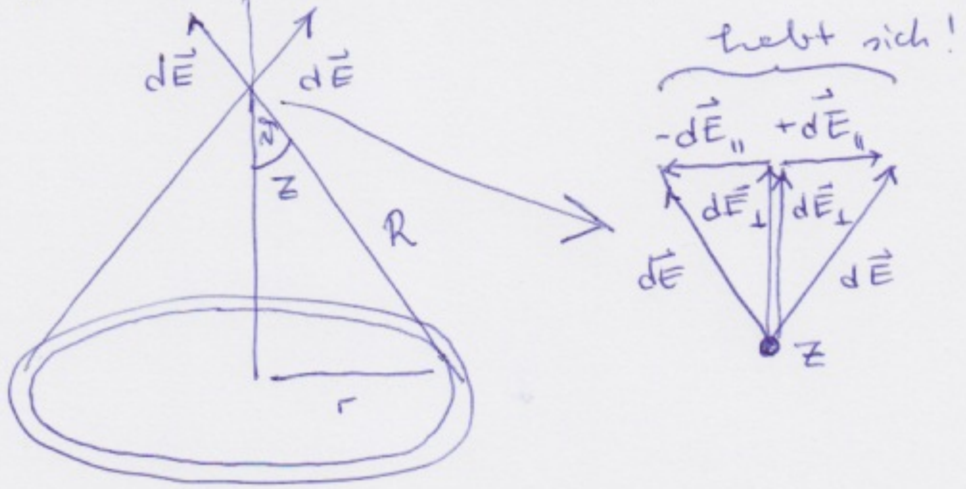
$$= \frac{\sigma}{2\varepsilon_0} \left(\lim_{r\to\infty} \sqrt{r^2+z^2} - z \right) \quad \circledast$$

$$\underline{\underline{\vec{E}}} = -\nabla\Phi = -\frac{\partial\Phi}{\partial z}\hat{z} \quad \text{denn laut } \circledast$$

sind die Äquipotentialflächen Ebenen $z = \text{const}$

$$\stackrel{\downarrow}{=} -\frac{\sigma}{2\varepsilon_0} \left(\lim_{r\to\infty} \cancel{\frac{2z}{2\sqrt{r^2+z^2}}} - 1 \right) \hat{z} = \underline{\underline{\frac{\sigma}{2\varepsilon_0} \hat{z}}}$$

Zweiter Weg: direkt $\vec{E}$ ausrechnen



$$\underline{\underline{\vec{E}(z)}} = \frac{1}{4\pi\varepsilon_0} \int_0^\infty \underbrace{\frac{\sigma\, 2\pi r\, dr}{r^2+z^2}}_{\frac{dq}{R^2}} \cdot \underbrace{\frac{z}{\sqrt{r^2+z^2}}}_{\frac{z}{R} = \cos\vartheta} \hat{z}$$

gibt nur Normalkomponente von $\vec{E}$ zur Platte.

$$= \frac{\sigma}{2\varepsilon_0} \hat{z}\, z \int_0^\infty dr\, \frac{r}{R^3}$$

$$\overset{\text{Angabe}}{=} \frac{\sigma}{2\varepsilon_0} \hat{z}\, z \left[-\frac{1}{R}\right]_{r=0}^{\infty}$$

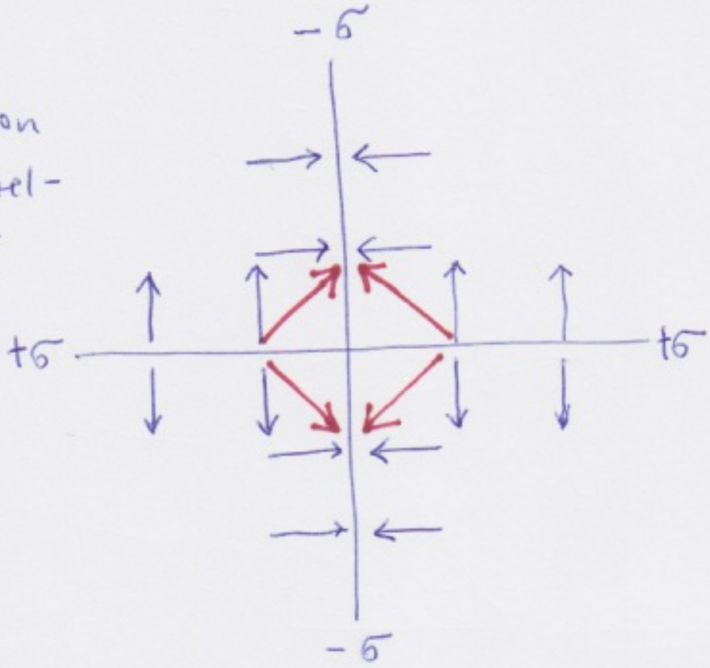
$$= \frac{\sigma}{2\varepsilon_0} \hat{z}\, z \left[-\frac{1}{\sqrt{r^2+z^2}}\right]_{r=0}^{\infty}$$

$$= \frac{\sigma}{2\varepsilon_0} \hat{z}\, z \left[-0 - -\frac{1}{z}\right] = \underline{\underline{\frac{\sigma}{2\varepsilon_0} \hat{z}}}$$

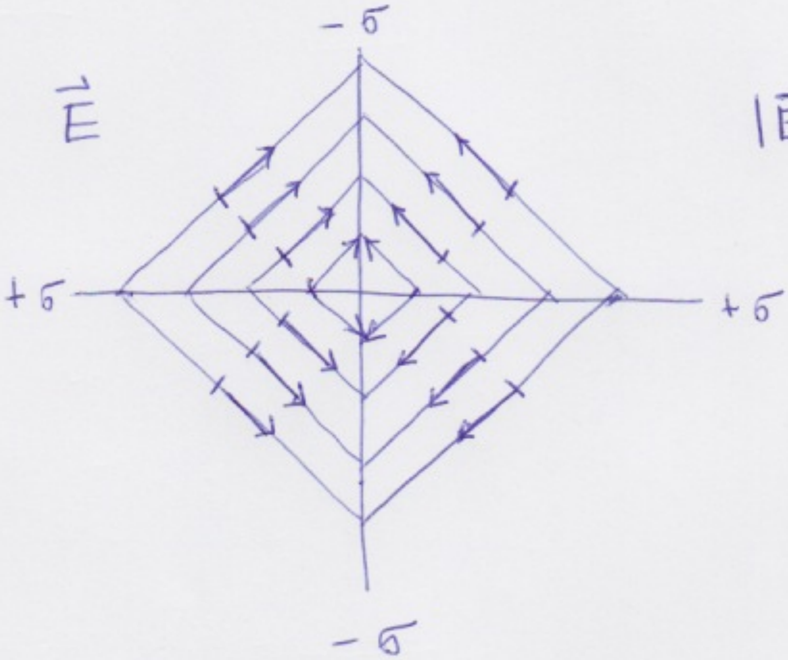
wie zuvor. Das elektrische Feld ist also konstant!

1 b

Addition der Einzelfelder



also $\vec{E}$



$|\vec{E}| = \text{const}!$

(2)

$$V(x) = \lim_{a\to 0} \frac{1}{4\pi\varepsilon_0} \left[\frac{q}{x+a} - \frac{q}{x} - \left(\frac{q}{x} - \frac{q}{x-a} \right) \right]$$

$$= \lim_{a\to 0} \frac{1}{4\pi\varepsilon_0} q \left[\frac{d}{dx}\frac{1}{x}\bigg|_{x+\frac{a}{2}} a - \frac{d}{dx}\frac{1}{x}\bigg|_{x-\frac{a}{2}} a \right]$$

$\left(f = \frac{d}{dx}\frac{1}{x}\right)$

$$= \lim_{a\to 0} \frac{qa}{4\pi\varepsilon_0} \left[f\left(x+\frac{a}{2}\right) - f\left(x-\frac{a}{2}\right) \right]$$

$$= \lim_{a\to 0} \frac{qa^2}{4\pi\varepsilon_0} \frac{df}{dx}\bigg|_x$$

$$= \lim_{a\to 0} \frac{qa^2}{4\pi\varepsilon_0} \frac{d}{dx}\left[\frac{d}{dx}\frac{1}{x}\right]\bigg|_x$$

$$= \lim_{a\to 0} \frac{qa^2}{4\pi\varepsilon_0} \frac{2}{x^3}$$

(3)

$$\vec{A} = \frac{\mu_0 I}{4\pi} \oint \frac{d\vec{r}\,'}{|\vec{r} - \vec{r}\,'|}$$

$$\approx \frac{\mu_0 I}{4\pi} \oint d\vec{r}\,' \left[\frac{1}{r} + \nabla \frac{1}{r} \cdot \left[(\vec{r} - \vec{r}\,') - \vec{r} \right] \right]$$

$$= \frac{\mu_0 I}{4\pi} \; \frac{1}{r} \underbrace{\oint d\vec{r}\,'}_{=0} - \oint d\vec{r}\,' \left(\nabla \frac{1}{r} \right) \cdot \vec{r}\,'$$

nur hier Skalarprodukt!

$$= \frac{\mu_0 I}{4\pi} \oint d\vec{r}\,' \left(\frac{\hat{r}}{r^3} \cdot \vec{r}\,' \right)$$

$$= \frac{\mu_0 I}{4\pi} \oint d\vec{r}\,' \left(\vec{r}\,' \cdot \frac{\hat{r}}{r^3} \right)$$

$$= \underbrace{\frac{\mu_0 I}{4\pi} \left(\oint d\vec{r}\,' \,|\, \vec{r}\,' \right)}_{\overleftrightarrow{M}} \cdot \frac{\hat{r}}{r^3}$$

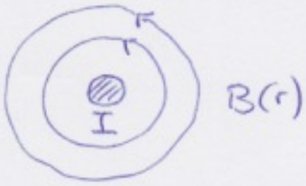
$$= \overleftrightarrow{M} \cdot \frac{\hat{r}}{r^3}$$

4 • $\vec{B}$ - Feld aus Ampere:

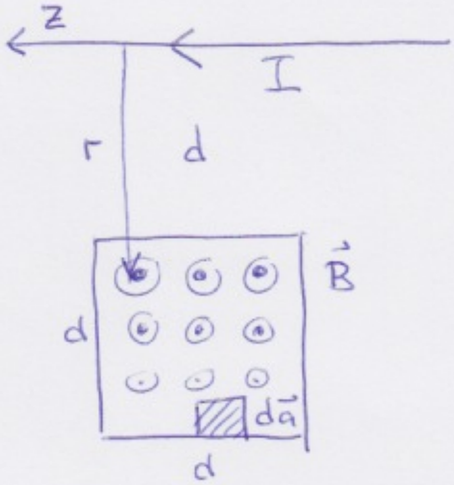
$$\text{rot } \vec{B} = \mu_0 \vec{J}$$, im geraden Leiter

$$2\pi r \, B(r) = \oint d\vec{l} \cdot \vec{B} \underset{\text{Stokes}}{=} \int d\vec{a} \cdot \text{rot } \vec{B} = \mu_0 \int d\vec{a} \cdot \vec{J} = \mu_0 I$$

also $$B(r) = \frac{\mu_0 I}{2\pi} \frac{1}{r}$$



• Faraday:



$$\text{EMF} = \oint \vec{E} \cdot d\vec{l} = -\dot{\Phi}$$

$$= -\frac{d}{dt} \int d\vec{a} \cdot \vec{B}$$

$$= -\frac{\mu_0}{2\pi} \int d\vec{a} \, \frac{dI}{dt} \frac{1}{r}$$

Interpretation: verwende Zylinderkoordinaten r, z in Integration

$$= -\frac{\mu_0}{2\pi} \underbrace{\int_d^{2d} dr \, \frac{1}{r} \int_0^d dz}_{da/r} \quad \frac{dI}{dt}$$

$$\text{EMF} = -\frac{\mu_0}{2\pi} \frac{dI}{dt} \ln 2 \cdot d$$

(5)

$$\omega = \omega_{kl}\, e^k \wedge e^l$$

$$d\omega = \frac{\partial \omega_{kl}}{\partial x_i}\, e^i \wedge e^k \wedge e^l$$

$$dd\omega = \frac{\partial^2 \omega_{kl}}{\partial x_i \partial x_j}\, e^i \wedge e^j \wedge e^k \wedge e^l$$

$$\underset{(*)}{=} \frac{\partial^2 \omega_{kl}}{\partial x_j \partial x_i}\, e^i \wedge e^j \wedge e^k \wedge e^l$$

$$\underset{(**)}{=} -\frac{\partial^2 \omega_{kl}}{\partial x_j \partial x_i}\, e^j \wedge e^i \wedge e^k \wedge e^l$$

$$\underset{(***)}{=} -\frac{\partial^2 \omega_{kl}}{\partial x_i \partial x_j}\, e^i \wedge e^j \wedge e^k \wedge e^l$$

$$= -\, dd\omega$$

d.h. $\quad 2\, dd\omega = 0$

d.h. $\quad \underline{\underline{dd\omega = 0}}$

(*) : $\dfrac{\partial^2}{\partial x \partial y} = \dfrac{\partial^2}{\partial y \partial x}$

(**) : $e^i \wedge e^j = -e^j \wedge e^i$

(***) : Umbenennen $i \leftrightarrow j$

6 $\rho(x,t) = e\,\delta(x - x_0(t))$

$$4\pi\varepsilon_0 \Phi(x,t) = \int dx' \frac{e\,\rho(x',t_r)}{|x-x'|} \quad \text{mit} \quad t_r \overset{(*)}{=} t - \frac{|x-x'|}{c}$$

$$= e \int dx' \frac{\delta(x' - x_0(t_r))}{|x-x'|}$$

Zitat: „Integral ausführen heißt $x' = x_0(t_r)$" (**)

$$\overset{(**)}{=} \frac{e}{|x - x_0(t_r)|}$$

$$\overset{(*)}{=} \frac{e}{\left|x - x_0\left(t - \frac{|x-x'|}{c}\right)\right|}$$

$$\overset{(**)}{=} \frac{e}{\left|x - x_0\left(t - \frac{x - x_0(t_r)}{c}\right)\right|}$$

$$\overset{(*)}{=} \frac{e}{\left|x - x_0\left(t - \frac{x - x_0\left(t - \frac{|x-x'|}{c}\right)}{c}\right)\right|}$$

$$\overset{(**)}{=} \frac{e}{\left|x - x_0\left(t - \frac{x - x_0\left(t - \frac{|x - x_0(t_r)|}{c}\right)}{c}\right)\right|}$$

$$\overset{(*)}{=} \frac{e}{\left|x - x_0\left(t - \frac{x - x_0\left(t - \frac{x - x_0\left(t - \frac{|x-x'|}{c}\right)}{c}\right)}{c}\right)\right|}$$