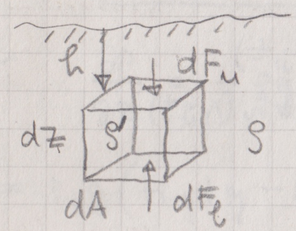
gelbe Karteikarten 4.4.22

Internal gravity waves

Buoyancy force

I)



$dF_u = \rho g h \, dA$ (Archimed.)

$dF_l = \rho g (h + dz) dA$

$dF_B = dF_u - dF_l$

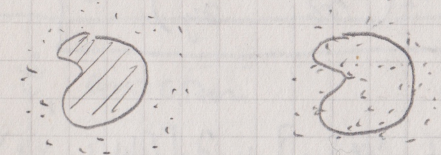
$= -\rho g \, dz \, dA$

$$g_B = \frac{dF_l}{dm} = -\frac{\rho \, dV}{\rho' \, dV} g = -\frac{\rho}{\rho'} g$$

if $\rho = \rho'$ then $g_l = -g$: equilibrium

if $\rho' < \rho$ then $\updownarrow^{g_l}_{g}$: body is driven upward

II) Archimedisches Prinzip

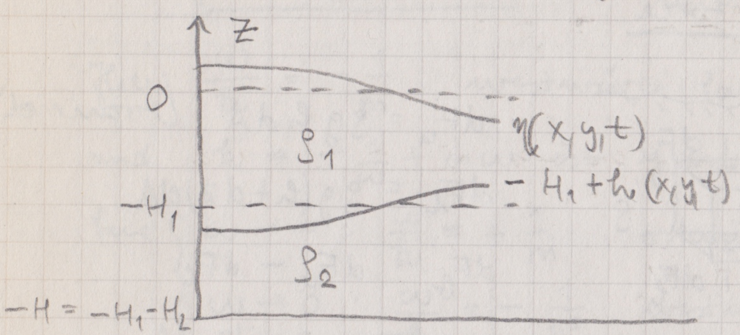


Pe in Water — Water in Water

the same F_l, since it is caused by outer water & doesn't know body interior.

Two superimposed layers — A. Gill
with internal gravity waves

use shallow water theory! Stokes 1847



$p_1 = \rho_1 g (\eta - z)$ aus $\frac{\partial p}{\partial z} = -\rho g$

Kontinuitätsgleichungen

unten

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}\Big[(H_2+\underbrace{h)u_2}_{\text{klein}}\Big] + \frac{\partial}{\partial y}\Big[(H_2+\underbrace{h)v_2}_{\text{klein}}\Big] = 0$$

$$\boxed{\frac{\partial h}{\partial t} + H_2\left(\frac{\partial u_2}{\partial x} + \frac{\partial v_2}{\partial y}\right) = 0} \qquad C_2$$

oben

$$\frac{\partial}{\partial t}(\eta - h) + \frac{\partial}{\partial x}\Big[(H_1 + \underbrace{\eta - h)u_1}_{\text{klein}}\Big] + \frac{\partial}{\partial y}\Big[(H_1 + \underbrace{\eta - h)v_1}_{\text{klein}}\Big] = 0$$

$$\boxed{\frac{\partial}{\partial t}(\eta - h) + H_1\left(\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y}\right) = 0} \qquad C_1$$

Eulergleichungen

oben

$$\boxed{\begin{aligned}\frac{\partial u_1}{\partial t} &= -g\frac{\partial \eta}{\partial x}\\ \frac{\partial v_1}{\partial t} &= -g\frac{\partial \eta}{\partial y}\end{aligned}} \qquad E_1$$

unten

$$\frac{\partial p}{\partial z} = -\rho g \rightarrow \Delta p = -\rho g\, \Delta z$$

$$p_2 = \rho_1 g(H_1 + \eta - h) + \rho_2 g(-H_1 + h - z)$$ einsetzen in

$$\frac{\partial u_2}{\partial t} = -\frac{1}{\rho_2}\frac{\partial p_2}{\partial x}$$ gibt

$$\boxed{\begin{aligned}\frac{\partial u_2}{\partial t} &= -\frac{\rho_1}{\rho_2} g \frac{\partial \eta}{\partial x} - \frac{\rho_2 - \rho_1}{\rho_2} g \frac{\partial h}{\partial x}\\ \frac{\partial v_2}{\partial t} &= -\frac{\rho_1}{\rho_2} g \frac{\partial \eta}{\partial y} - \frac{\rho_2 - \rho_1}{\rho_2} g \frac{\partial h}{\partial y}\end{aligned}} \qquad E_2$$

E_1 in $\dot{C}_1$ einsetzen

$$\frac{\partial^2}{\partial t^2}(\eta - h) + H_1\left(-g\frac{\partial^2 \eta}{\partial x^2} - g\frac{\partial^2 \eta}{\partial y^2}\right) = 0$$

(*) $$\boxed{\frac{\partial^2}{\partial t^2}(\eta - h) - g H_1 \Delta_h \eta = 0} \qquad \Delta_h = \partial_x^2 + \partial_y^2$$

E_2 in $\dot{C}_2$ einsetzen

$$\frac{\partial^2 h}{\partial t^2} + H_2\left(-\frac{\rho_1}{\rho_2} g \frac{\partial^2 \eta}{\partial x^2} - \frac{\rho_2-\rho_1}{\rho_2} g \frac{\partial^2 h}{\partial x^2} - \frac{\rho_1}{\rho_2} g \frac{\partial^2 \eta}{\partial y^2} - \frac{\rho_2-\rho_1}{\rho_2} g \frac{\partial^2 h}{\partial y^2}\right) = 0$$

$$\boxed{\frac{\partial^2 h}{\partial t^2} - g H_2 \Delta_h \left(\frac{\rho_1}{\rho_2}\eta + \frac{\rho_2-\rho_1}{\rho_2} h\right) = 0} \qquad (**)$$

(*), (**) sind 2 Glg 2. Ordnung für η & h, also
1 Glg 4. Ordnung für η

Vereinfachende Annahme: (A) Synchronität

$$\boxed{h = \mu \eta} \qquad \mu = \text{const}$$

$$\left.\begin{aligned} &\frac{\partial^2}{\partial t^2}\eta - \frac{g H_1}{1-\mu}\Delta_h \eta = 0 \\ &\frac{\partial^2}{\partial t^2}\eta - \frac{g H_2}{\mu}\Delta_h\left(\frac{\rho_1}{\rho_2} + \frac{\rho_2-\rho_1}{\rho_2}\mu\right)\eta = 0 \end{aligned}\right\} \oplus$$

also muss sein

$$\frac{g H_1}{1-\mu} = \frac{g H_2}{\mu}\left(\frac{\rho_1}{\rho_2} + \mu \frac{\rho_2-\rho_1}{\rho_2}\right)$$

$$= \frac{g H_2}{\mu}\left(\frac{\rho_1-\rho_2}{\rho_2} + 1 + \mu\frac{\rho_2-\rho_1}{\rho_2}\right) \quad \text{also}$$

$$\frac{g H_1}{1-\mu} = \frac{g H_2}{\mu}\left(1-(1-\mu)\varkappa\right) \quad \text{mit } \varkappa = \frac{\rho_2-\rho_1}{\rho_2}$$

Einsetzen in $\oplus$

$$\ddot\eta - c^2 \Delta_h \eta = 0 \quad \text{mit } c^2 = \frac{g H_1}{1-\mu} = \frac{g H_2}{\mu}\left(1-(1-\mu)\varkappa\right)$$

Eliminiere μ: $\quad 1-\mu = \frac{g H_1}{c^2}, \; \mu = 1 - \frac{g H_1}{c^2}$

dann $\quad \left(1 - \frac{g H_1}{c^2}\right)c^2 = g H_2\left(1 - \varkappa \frac{g H_1}{c^2}\right)$

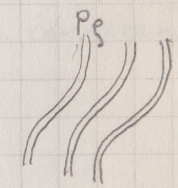
$$c^2 - g(H_1+H_2) + \varkappa g^2 H_1 H_2 / c^2 = 0$$

$$\boxed{c^4 - g H c^2 + \varkappa g^2 H_1 H_2 = 0} \qquad H = H_1 + H_2$$

$$c^2 = \frac{gH}{2} \pm \frac{gH}{2}\sqrt{1 - 4\varkappa H_1 H_2 / H^2}$$

Für $\varkappa \ll 1$:

$$c^2 = \frac{gH}{2} \pm \frac{gH}{2} \mp g H \varkappa H_1 H_2 / H^2$$



Normal mode = barotropic, $p = p(\rho)$

$$\boxed{c^2 = gH} \cdot \left(1 - \varkappa H_1 H_2 / H^2\right)$$

und $\quad \frac{\eta}{h} = \frac{1}{\mu} = \frac{1}{1 - g H_1 / c^2} = \frac{1}{1 - \frac{g H_1}{g(H_1+H_2)}}$

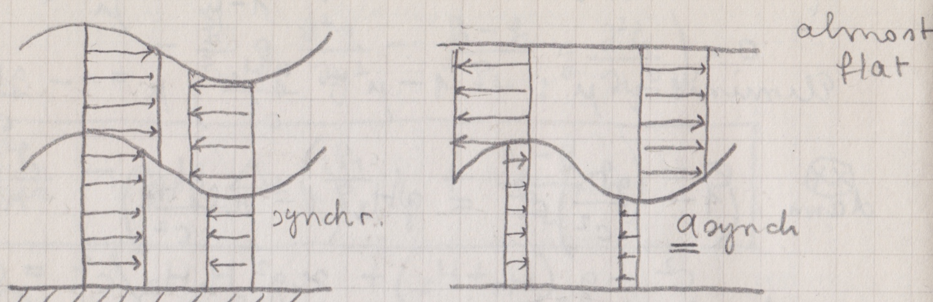
also $\quad \boxed{\frac{\eta}{h} = \frac{H_1+H_2}{H_2}} \quad$ = surface gravity wave

Baroclinic mode

$$\boxed{c^2 = g H \varkappa H_1 H_2 / H^2}$$ ist langsam wegen $\varkappa \ll 1$.

und $\quad \frac{\eta}{h} = \frac{1}{\mu} = \frac{1}{1 - \underbrace{\frac{g H_1 H^2}{g H \varkappa H_1 H_2}}_{\gg 1}}, \quad \boxed{\frac{\eta}{h} \approx -\varkappa \frac{H_2}{H_1+H_2}}$

baroclinic mode = internal gravity wave



barotropic surface wave

$c = \sqrt{gH}$

baroclinic internal wave

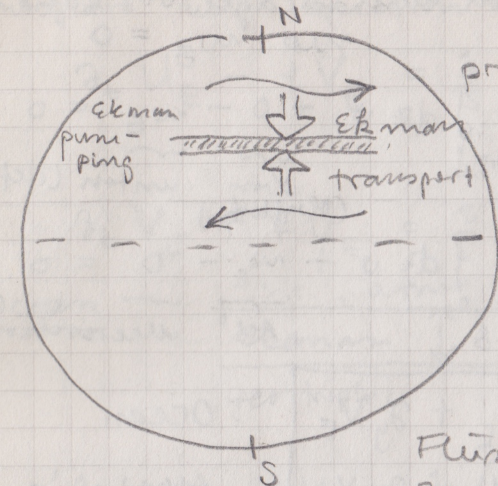
$c \ll \sqrt{gH}$

Ekman Pumping

7.4.22

Gill S. 326, 327

= vertical flow, induced by horizontal Ekman transport.



preferrential wind direction („westerly")

trade winds („easterly")

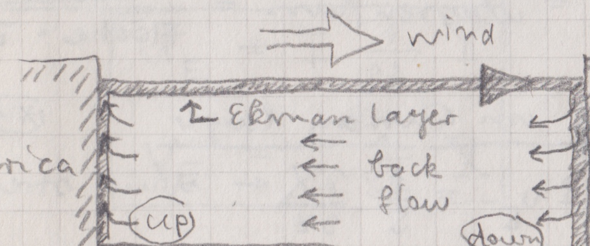
convergence of mass

Flüssigkeit muss senkrecht zum Gradienten des Windes fließen

Horizontal ist nicht länger möglich wg. convergence

⇒ Vertical transport

Oder so:



= Friction dynamics in three thin boundary layers

central result: $\boxed{\rho w_E = \frac{1}{f}\left(\partial_x Y - \partial_y X\right)}$

Kontinuitätsgleichung

$$u_x + v_y + w_z = 0$$

Integriere vertikal durch Ekmanlayer bis Rand wo $w_E = 0$

Ocean: $\partial_x \int_{-\delta}^{0} dz\, u + \partial_y \int_{-\delta}^{0} dz\, v + 0 - w_E = 0$

(0: oben (Oberfläche); w_E: unten (Tiefsee))

Atmosphäre: $\partial_x \int_{0}^{\delta} dz\, u + \partial_y \int_{0}^{\delta} dz\, v + w_E - 0 = 0$

(w_E: „All"; 0: Meeresoberfläche)

also

$$(*)\quad \boxed{\begin{aligned} w_E &= \partial_x U_E + \partial_y V_E && \text{Ocean} \\ w_E &= -\partial_x U_E - \partial_y V_E && \text{Atmosphäre} \end{aligned}}$$

mit $U_E = \int dz\, u$: Massenfluss

denn $\rho \cdot u \cdot A = \frac{kg}{m^3} \cdot \frac{m}{s} \cdot m^2 =$ Massenstrom

also $\rho \cdot u =$ Massenfluss $= \frac{\text{Masse}}{\text{Fläche} \cdot \text{Zeit}}$

Eulergleichung im Ekman layer

$$\dot{u}_E - f v_E = \frac{1}{\rho} \frac{\partial X}{\partial z} \leftarrow \text{wind stress}$$

$$\dot{v}_E + f u_E = \frac{1}{\rho} \frac{\partial Y}{\partial z}$$

$u_E = v_E = X = Y = 0$ an Unter (Ocean) Kante / Ober (Atmo)

wo geostrophe Strömung

Atmosphäre

geostrophic / Ekman $\updownarrow \delta$ } atmosph. / (Ocean)

Integrate Euler equation vertically

$$\partial_t U_E - f V_E = \frac{1}{\rho}(0 - X) \quad \left(= \int_0^\delta dz\, X_z\right)$$

(0: „space"; X: shear on ocean)

$$\partial_t V_E + f U_E = \frac{1}{\rho}(0 - Y)$$

Ocean

atmo / Ekman $\updownarrow \delta$ / geostrophic } ocean

$$\partial_t U_E - f V_E = \frac{1}{\rho}(X - 0) \quad \left(= \int_{-\delta}^{0} dz\, X_z\right)$$

$$\partial_t V_E + f U_E = \frac{1}{\rho}(Y - 0)$$

(Y: ocean surface; 0: deep down)

assume stationary flow

$$(**)\quad \boxed{\begin{aligned} U_E &= \mp \frac{1}{\rho f} Y \\ V_E &= \pm \frac{1}{\rho f} X \end{aligned}}$$

upper sign: atmosphere

lower sign: ocean

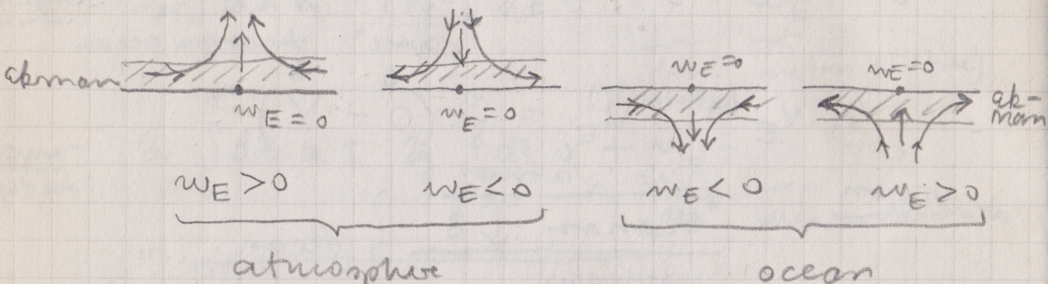
Combine (*) & (**) with $\rho = $ const, $f = $ const

$$\boxed{w_E = \frac{1}{\rho f}(\partial_x Y - \partial_y X)}$$

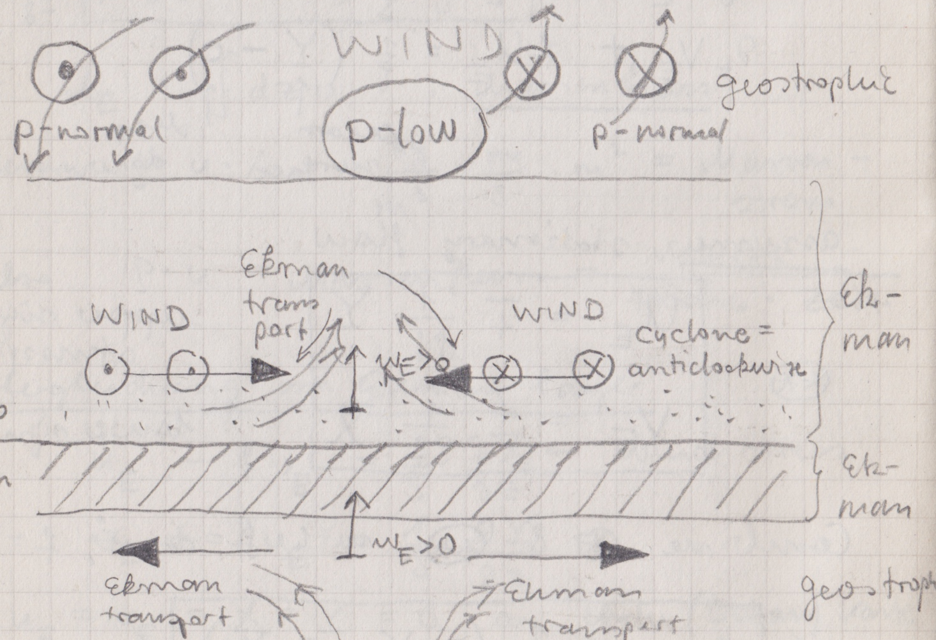
for both ocean & atmosphere! No sign difference

Thus: vertical ekman pumping
= curl of (horizontal) wind stress !

again for sign:

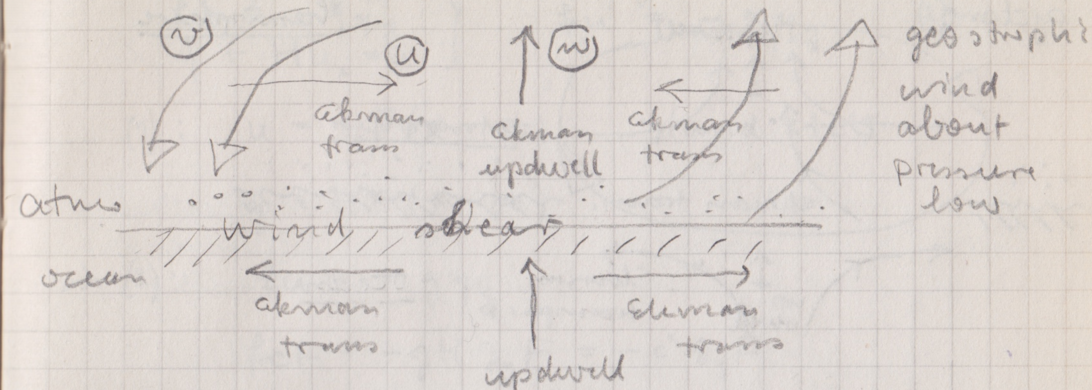


Cyclone over ocean



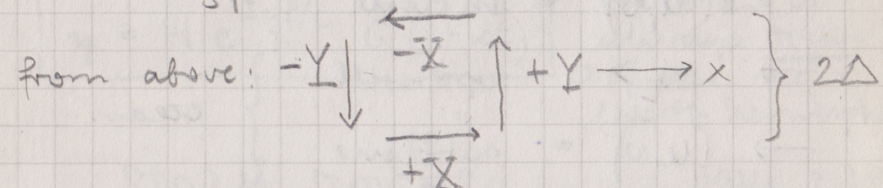
similarly for V_E & X →

thus 3-D velocity field



① discussion of w_E = ekman updwell

$$w_E = \frac{1}{\rho f}\left(\partial_x Y - \partial_y X\right)$$ atmosph & ocean

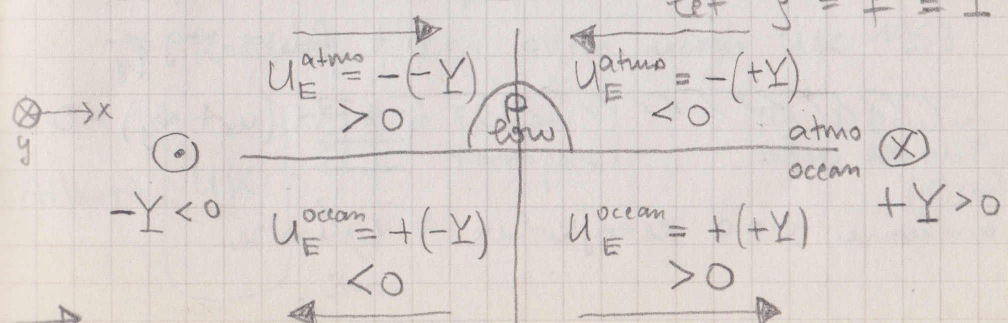


$$\partial_x Y = \frac{Y-(-Y)}{2\Delta} > 0$$

$$\partial_y X = \frac{-X - X}{2\Delta} < 0$$

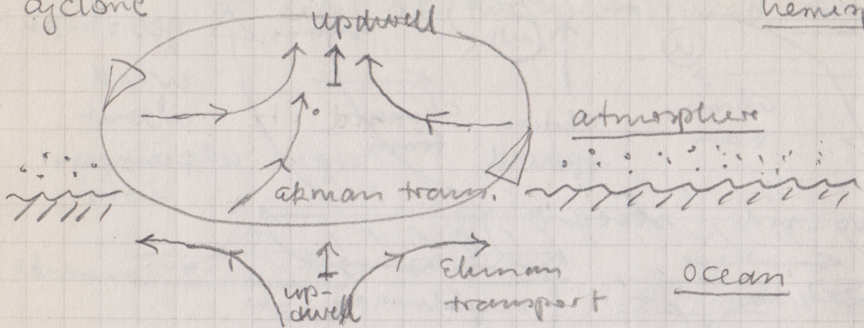
$w_E > 0$ for atmo & ocean

② discussion of u_E, v_E = Ekman transport
let $\rho = f = 1$



Summary

pressure low over ocean in north hemisphere



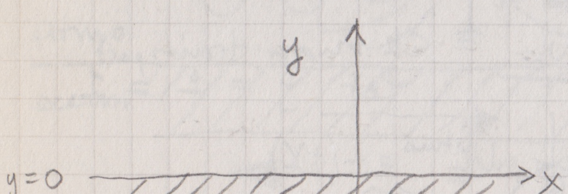
cyclone, positive vorticity

$\longrightarrow$ $w_E > 0$ updwell } atmosphere
$\longrightarrow$ (u,v) = inflow

$\longrightarrow$ $w_E > 0$ updwell } ocean
$\longrightarrow$ (u,v) = outflow

Remembrance for coming things:

Kelvin wave (Gill chap. 10)



$$\dot{u} - fv = -g\eta_x$$
$$\dot{v} + fu = -g\eta_y$$
$$\dot{\eta} + H(u_x + v_y) = 0$$

assume $v = 0$ everywhere & linearize

$$\left.\begin{array}{l} \dot{u} = -g\eta_x \\ fu = -g\eta_y \\ \dot{\eta} + Hu_x = 0 \end{array}\right\} \qquad \begin{array}{ll} \ddot{u} - gH\,u_{xx} = 0 & (1) \\ fu = -g\eta_y & (2) \end{array}$$

(1): $u = cF(x-ct)$ (and $x+ct$)

$\eta = HF(x-ct)$

in (2): $fu = -g\eta_y$

$fcF = -gH\,\partial_y F = -c^2\,\partial_y F$

$\frac{dF}{F} = -\frac{f}{c}\,dy$, $F = e^{-yf/c}$

$$\boxed{\begin{array}{l} u = c\,e^{-yf/c}\,G(x-ct) \\ \eta = H\,e^{-yf/c}\,G(x-ct) \end{array}}$$

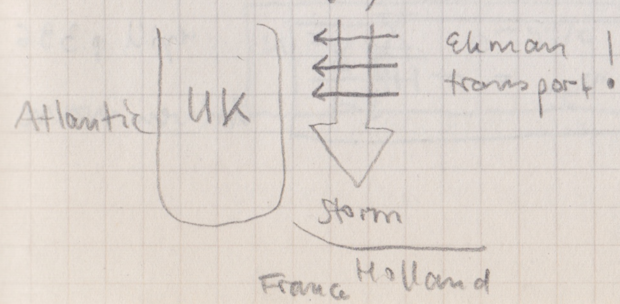
Kelvin wave

The coast is always right of the wave (North hemisphere)

STORM SURGES

Gill S. 395

Summary: a storm along a coast can cause stationary inflow normal to the coast, which causes $\eta \sim t$ of the water surface → flooding, e.g. storm surge UK 1953.

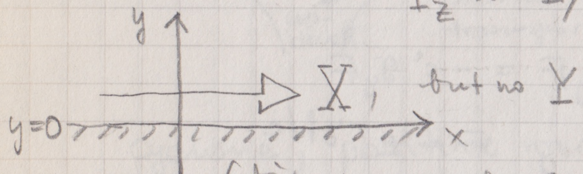


The following after Nomitsu (1934)

$$\dot u - f v = -g\eta_x + \frac{1}{\rho} X_z$$
$$\dot v + f u = -g\eta_y + \frac{1}{\rho} Y_z$$
$$\dot\eta + H(u_x + v_y) = 0$$

Assumptions: $X_z \approx X/H$ for shallow ocean near a coast

$Y_z \approx Y/H$



(bit strange for england...)

Furthermore: $\partial_x \equiv 0$. Then

$$(1)\quad \dot u - f v = X/\rho H \quad \leftarrow \text{Ekman transport}$$
$$(2)\quad \dot v + f u = -g\eta_y \quad \leftarrow \text{Kelvin welle}$$
$$(3)\quad \dot\eta + H v_y = 0$$

$-f(1) + \partial_t(2) - g\partial_y(3)$ gives

$$-f\dot u + f^2 v = -f\frac{X}{\rho H}$$
$$+\ \ddot v + f\dot u = -g\dot\eta_y$$
$$-g\dot\eta_y - gH v_{yy} = 0$$

$$\boxed{\ddot v + f^2 v - gH v_{yy} = -\frac{fX}{\rho H}}$$

Gill p. 396
driven wave

Solution is transient wave + stationary part

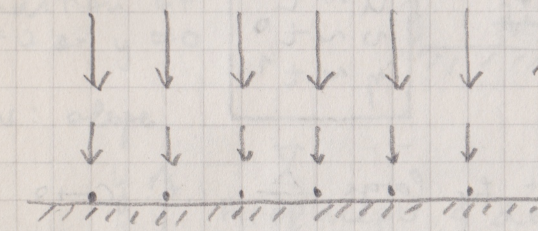
Neglect transient, consider only $\partial_t \equiv 0$

$$f^2 v - gH v_{yy} = -\frac{fX}{\rho H}$$

has solution

$$\boxed{v = -\frac{X}{f\rho H}\left(1 - e^{-yf/c}\right)} \qquad (c^2 = gH)$$

(check is simple)



$v \sim -(1 - e^{-y})$

Nomitsu (1934)

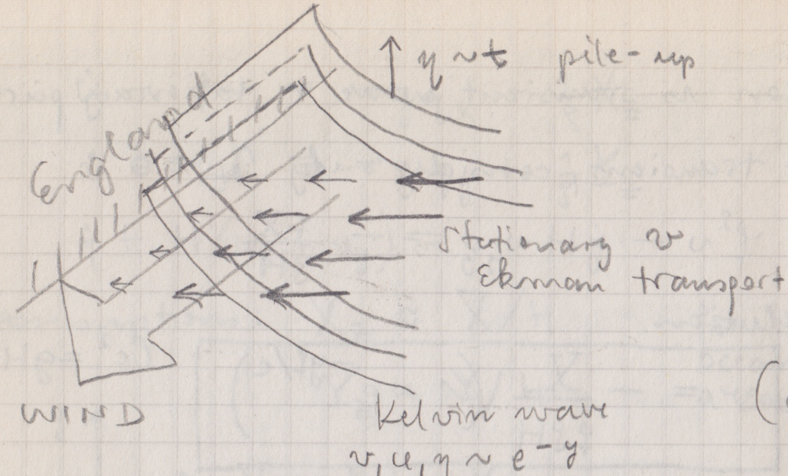
Continuity: $\dot\eta = -H v_y$

$$= -H\left(-\frac{X}{f\rho H}\right)\left(-\,-\frac{f}{c}e^{-yf/c}\right)$$
$$= +\frac{X}{\rho c}e^{-yf/c}$$

thus $\boxed{\eta(t) = \frac{X}{\rho c}e^{-yf/c}\cdot t}$ linear pile-up!

Euler: $\dot v + f u = -g\partial_y\eta$

$$\boxed{u = \frac{X}{\rho H}e^{-yf/c}\cdot t}$$



(also Hough 1897)

very rare result:

$$\boxed{\begin{array}{l} u \sim t^1 \\ v \sim t^0 \\ \eta \sim t^1 \end{array}}$$

affect is strongest for large $\frac{X}{gc}$, i.e. $c \to 0$

i.e. $H \to 0$ [Coast]

e.g. $H = 50\,m$, $X = 2\,N/m^2 \to$

$d\eta/dt = 1m/3h \;\hat{=}\;$ storm surge of 1953

Summary cause for flooding of coast is wind induced Ekman transport

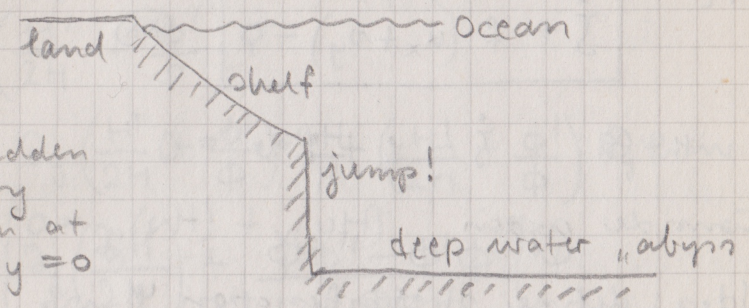
~ SHELF WAVES

Gill p. 409

So far $H \approx$ const. Now: strong change in H

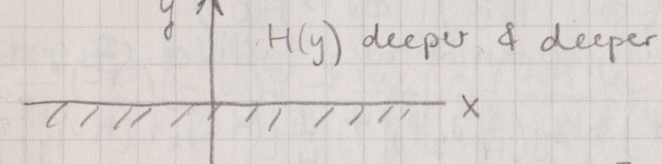
Ocean slope near coasts

Shelf:



so far: sudden boundary condition at $x = 0$ or $y = 0$

Now: slope



Continuity $\dot\eta + [(H+\eta)u]_x + [(H+\eta)v]_y = 0$

assumption $\eta \ll H$

$$\dot\eta + (Hu)_x + (Hv)_y = 0$$

rigid lid at top: $\dot\eta = 0$ (approximation!)

& with $H = H(y)$ alone $\boxed{u_x + v_y = -\frac{H_y}{H} v}$ ⊛

Shallow water

$$\dot u - fv = -g\eta_x$$

$$\dot v + fu = -g\eta_y$$

Vorticity $\zeta = v_x - u_y$

$$\dot{v}_x + f u_x + g \eta_{yx}$$
$$- \dot{u}_y + f v_y - g \eta_{xy} = 0$$

i.e.
$$\boxed{\dot{\zeta} + f (u_x + v_y) = 0} \quad (**)$$

with (*): $\dot{\zeta} - f \frac{H_y}{H} v = 0$

Consider again $(Hu)_x + (Hv)_y = 0$

Idea: define streamfunction ψ via

$$Hu = -\psi_y \,, \quad Hv = \psi_x$$

then continuity automatically fulfilled. (**) gives

$$0 = \partial_t \left[\partial_x \left(\frac{1}{H} \partial_x \psi \right) + \partial_y \left(\frac{1}{H} \partial_y \psi \right) \right] + \frac{f}{H} \left[(Hu)_x + (Hv)_y \right] - \frac{f}{H} H_x u - \frac{f}{H} H_y v$$

(product rule) (is 0)

$$\boxed{\partial_t \left[\partial_x \left(\frac{1}{H} \partial_x \psi \right) + \partial_y \left(\frac{1}{H} \partial_y \psi \right) - \frac{f}{H^2} \partial_y H \, \partial_x \psi = 0 \right.}$$

Gill (10.12.4)

has solution

$$\boxed{\psi(x, y, t) = \sqrt{H(y)} \, \phi(y) \, e^{i(kx - \omega t)}}$$

where ϕ obeys a new, simpler DGL.

Trick: leave $\frac{H'}{2H}$ wherever possible

$$\frac{1}{H} \partial_t \partial_x^2 \psi + \frac{1}{H} \partial_t \partial_y^2 \psi - \frac{\partial_y H}{H^2} \partial_t \partial_y \psi - \frac{f}{H} \frac{\partial_y H}{H} \partial_x \psi = 0 \quad (*)$$

$$\partial_y \psi = \frac{\partial_y H}{2\sqrt{H}^2} \left(\sqrt{H} \phi e^{\cdots} \right) + \frac{\partial_y \phi}{\phi} \left(\sqrt{H} \phi e^{\cdots} \right)$$
$$= \left(\frac{H'}{2H} + \frac{\phi'}{\phi} \right) \psi$$

$$\partial_y \partial_y \psi = \partial_y \left(\frac{H'}{2H} + \frac{\phi'}{\phi} \right) \cdot \psi + \left(\frac{H'}{2H} + \frac{\phi'}{\phi} \right) \partial_y \psi$$
$$= \left[\partial_y \left(\frac{\partial_y H}{2H} \right) + \frac{\phi \phi'' - \phi' \phi'}{\phi^2} \right] \psi$$
$$+ \left(\frac{\partial_y H}{2H} + \frac{\phi'}{\phi} \right) \left(\frac{\partial_y H}{2H} + \frac{\phi'}{\phi} \right) \psi$$

in (*)

$$\frac{1}{H} (-k^2)(-i\omega) \psi - \frac{i\omega}{H} \left[\partial_y \left(\frac{\partial_y H}{2H} \right) + \frac{\phi''}{\phi} - \left(\frac{\phi'}{\phi} \right)^2 + \left(\frac{\partial_y H}{2H} \right)^2 + \frac{\partial_y H}{H} \frac{\phi'}{\phi} + \left(\frac{\phi'}{\phi} \right)^2 \right] \psi$$
$$- \frac{2}{2} \frac{\partial_y H}{H H} (-i\omega) \left(\frac{H'}{2H} + \frac{\phi'}{\phi} \right) \psi - \frac{f}{H} \frac{\partial_y H}{H} ik \psi = 0$$

$$- \frac{\omega}{H} \frac{\phi''}{\phi} - \frac{\omega}{H} \partial_y \left(\frac{\partial_y H}{2H} \right) - \frac{\omega}{H} \left(\frac{\partial_y H}{2H} \right)^2$$
$$+ 2 \frac{\omega}{H} \left(\frac{\partial_y H}{2H} \right)^2 + \frac{k^2 \omega}{H} - \frac{fk}{H} \frac{\partial_y H}{H}$$
$$= 0 \quad \Big| \cdot - \frac{H\phi}{\omega}$$

(as so often)

$$\boxed{\phi'' + \phi \left\{ \left(\frac{H'}{2H} \right)' - \left(\frac{H'}{2H} \right)^2 - k^2 + \frac{fk}{\omega} \frac{H'}{H} \right\} = 0}$$

Gill p. 409 (10.12.6), roughly similar Pedlo p. 548

Specific shelf profile: $\boxed{H = H_0 e^{2\lambda y}}$

gives $-\ell^2\phi + 0\phi - \lambda^2\phi - k^2\phi + \frac{fk}{\omega}2\lambda = 0$

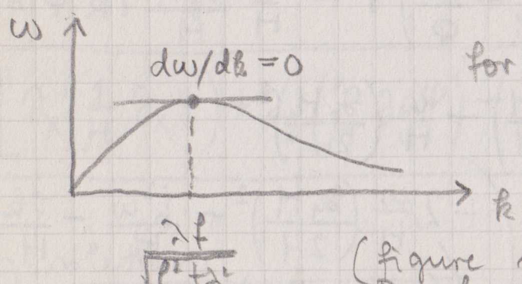
$$\boxed{\omega = \frac{2\lambda f k}{k^2+\ell^2+\lambda^2}}$$ Gill 10.12.9

Very similar to Rossby wave dispersion relation, for similar physics: potential vorticity conservation, see below.

$$0 = \frac{d\omega}{dk} = \left(k^2+\ell^2+\lambda^2\right)2\lambda f - 2\lambda f k\, 2k \;/\ldots$$
$$= \left(\ell^2+\lambda^2-k^2\right)2\lambda f\;/\ldots$$

i.e. group speed zero if $k^2 = \ell^2+\lambda^2$

i.e. $\omega = \frac{2\lambda f\sqrt{\ell^2+\lambda^2}}{2(\ell^2+\lambda^2)} = \frac{\lambda f}{\sqrt{\ell^2+\lambda^2}}$

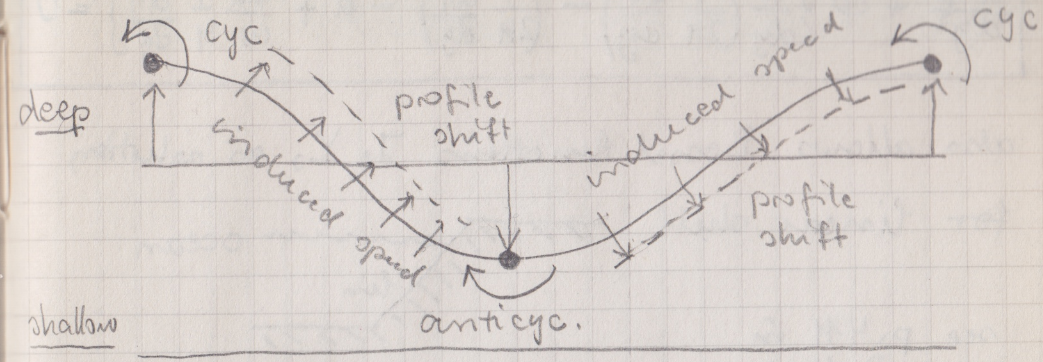


for short waves $(k \to \infty)$
$\mathrm{sgn}(v_\phi) = -\mathrm{sgn}(v_g)$

(figure similar to that for Rossby waves except for global sign $\omega = -\ldots$)

for $v_g = 0$: Leewaves (mountain waves) due to topography

Physical mechanism as before



1) consider linear – straight vortex chain
2) sinusoidal perturbation causes vorticity change
$$V = \frac{f+J}{H} = \text{const} \quad \text{(pot. vort.)}$$
i.e. if vortex goes deeper, $H\uparrow$, also $J\uparrow$: cyclonic,
if vortex goes shallower, $H\downarrow$, also $J\downarrow$: anti-cyclonic
3) these cyclonic & anticyclonic changes in J cause a velocity field
4) that shifts the sinusoidal („phase") profile of perturbation to the right

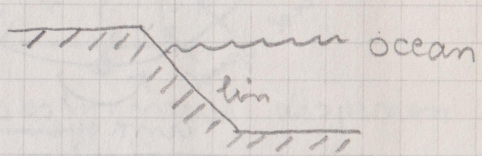
→ phase propagation with the coast & shallower water to the right in the north hemisphere, as usual.

the differential equation

$$\frac{d^2\phi}{dy^2} + \phi\left\{\frac{d}{dy}\left(\frac{1}{2H}\frac{dH}{dy}\right) - \left(\frac{1}{2H}\frac{dH}{dy}\right)^2 - k^2 + \frac{fk}{\omega}\frac{1}{H}\frac{dH}{dy}\right\} = 0$$

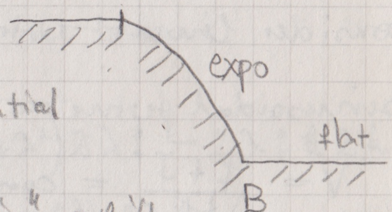
also allows Bessel functions J_0, Y_0 as solution,

for linear shelf



see p. 411 in Pedlosky

Other shelf profile:



turnover from exponential to flat

„east coast of Australia" → Gill